Macro 1: Tutorial 4*

Random Matching, Bargaining, and Match-Specific Productivity

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This tutorial introduces an interesting, useful and realistic extension of the standard model of employment fluctuations, which you were introduced to in lectures (and which followed the treatment of the standard model by Mortensen and Nagypál, 2007, *Review of Economic Dynamics*). The extension allows for match-specific productivity. Workers, who are all identical when unemployed, can now have different levels of productivity and wages when matched with a firm. This adds a new decision for the workers and firms: after have they matched, should they accept the match, given some randomly drawn level of match-specific productivity, or should they continue to search?

The treatment of the model here attempts to follow the notation and structure from the lectures, but you can also see the set-up and discussion thereof in Rogerson, Shimer & Wright, 2005, *Journal of Economic Literature*: Section 4.4 and Pissarides, 2000: Chapter 6.

In what follows, I describe the model preliminaries/set-up. Then in ***italics*** I suggest several exam-type questions which could follow such a set-up, so that you can later practise answering a question on this type of model. I give the algebraic solutions. You can later attempt the derivations and discussion yourselves using the literature cited above and what we discuss in the tutorial.

Model preliminaries:

- Time is continuous
- Workers are risk neutral (linear preferences) and discount the future at rate r.
- There is a mass 1 of workers who always participate in the labour market.
- An employed worker produces py in every period. $p > 0$ gives the aggregate level of productivity, common to all matches. Whilst it is trivial to set the model up such that p is stochastic and drawn at some rate from some known distribution (see lectures), here we assume p is a parameter. y is match-specific, and is drawn *after* a firm and a worker match in any period from known distribution (cdf) $F(y)$ with support $y \in [0, \infty]$.
- Unemployed workers earn a flow value of $z \geq 0$ and firms post vacancies at flow cost $c > 0$.

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- Matches are exogenously destroyed at rate $s > 0$.
- The matching process between the mass of unemployed workers u and mass of open vacancies v is given by $m(u,v)$, which has constant returns to scale, and is increasing and concave in both arguments.
- If a worker and firm decide to start producing together, they will do so at the same level of y forever unless the match is exogenously destroyed.
- The job-finding rate for unemployed workers is given by $f(\theta)$ where $\theta = v/u$. Vacancies meet workers at rate $q(\theta) = f(\theta)/\theta$: note, this is no longer "the job filling rate".
- The wage is given by standard "augmented" Nash bargaining solution

$$w(y) \in argmax\,[W(y) - U]^{\beta}\,[J(y)]^{1-\beta} \ , \tag{1}$$

  where $\beta \in [0,1]$ gives the worker's relative bargaining power. $W(y)$ and U are a worker's value of being employed and unemployed respectively. $J(y)$ gives the firm's value of employing a worker. The value of a firm holding an open vacancy is given by $V = 0$, where the latter equality results from a free-entry condition on opening vacancies.
- <u>Deviation from the baseline model in lectures</u>: Although all workers and firms are ex ante identical before meeting, only matches which draw a productivity greater than some reservation level $y \geq y_R$ will be accepted - not all meetings between workers and firms result in employment. You can assume that both workers and firms have the same reservation level.

1. Write-down Bellman equations of an employed and unemployed worker. Discuss the economic meaning of each and how they differ from the baseline model you saw in lectures.

An employed worker's Bellman equation is given by:

$$rW(y) = w(y) + s\,[U - W(y)] \ . \tag{2}$$

An unemployed worker's Bellman equation is given by:

$$rU = z + f(\theta)\int_0^{\infty} \max\{[W(x) - U]\,,0\}dF(x) \ .$$

or

$$rU = z + f(\theta)\int_{y_R}^{\infty} [W(x) - U]\,dF(x) \ . \tag{3}$$

2. Write-down the Bellman equations of a firm which employs a single worker and one which holds a single open vacancy. Discuss the economic meaning of each and how they differ from the baseline model you saw in lectures.

The value to a firm of employing a worker is given by:

$$rJ(y) = py - w(y) - sJ(y) \ . \tag{4}$$

The value to a firm of an open vacancy is given by:

$$rV = -c + q(\theta) \int_0^\infty \max\{J(x), 0\} dF(x) \quad (= 0) \ .$$

or

$$rV = -c + q(\theta) \int_{y_R}^\infty J(x) dF(x) \quad (= 0) \ . \tag{5}$$

3. Define the surplus of a match as $S(y) = J(y) + W(y) - U$. Write the surplus sharing rules from the Nash bargaining solution in terms of $S(y)$. Using this, discuss why the assumption that both firms and workers have the same reservation productivity policy is correct.

$$W(y) - U = \beta S(y) \ . \tag{6}$$

$$J(y) = (1-\beta) S(y) \ . \tag{7}$$

4. Use your answers to 1-3 to find an expression for y_R which only depends on θ and model parameters. Discuss this expression.

First using (2) and (4) find that

$$S(y) = \frac{py - rU}{r+s} \ . \tag{8}$$

Next using (3), (4) and (6) find an expression for rU and substitute into (8). This allows us to see that

$$y_R = \frac{z}{p} + \frac{\beta c \theta}{p(1-\beta)} \tag{9}$$

5. Using your answer to 4. and the free-entry condition, show the following:

$$(r+s)c = q(\theta)(1-\beta) p \int_{y_R}^\infty [x - y_R] \, dF(x) \tag{10}$$

6. The answers to 4. and 5. represent part of the equilibrium of this labour market. Discuss comparative statics (e.g. with respect to z, p and a mean-preserving spread in match-specific productivity draws.).

7. To complete the characterisation of the equilibrium, find expressions for the following:

- The wage $w(y)$ and it's average value $E[w(y)]$, which depend on θ, y and model parameters.
- The evolution of unemployment $\dot{u}$, which depends on θ and y_R.
- The evolution of the vacancy rate $\dot{v}$, which depends on θ and y_R and model parameters.
- The steady state values u^{ss} and v^{ss}, which depend on θ and y_R and model parameters.

$$w(y) = (1-\beta)z + \beta p(y + c\theta)\ . \tag{11}$$

$$E[w(y)] = (1-\beta)z + \beta p\left(\int_{y_R}^{\infty} x dF(x) + c\theta\right)\ . \tag{12}$$

$$\dot{u} = (1-u)s - f(\theta)[1-F(y_R)]u \tag{13}$$

$$\dot{u} = \dot{v} \tag{14}$$

$$u = \frac{s}{s + f(\theta)[1-F(y_R)]} \tag{15}$$

$$v = u\theta \tag{16}$$

8. Use the complete characterisation of the equilibrium you have now derived to discuss the effect of a positive and permanent aggregate shock to productivity $p' > p$ on wages, unemployment and vacancies. You should assume the labour market was initially at its stead-state before the arrival of the shock. You should use diagrams to explain your answer where appropriate.