

# Macro 1: Revision Tutorial\*

Extra Problem

17 April, 2017

This is an extra problem that I hope you find useful in your revision. I will present solutions in the April revision tutorial.

## Micro Foundations of Learning-by-doing and Fiscal Policy

*Note similarities to Macro 1 Degree Exam 2012/13 Q4 and Romer problem 3.11 (4th edition only)*

Suppose that firm  $i$ 's output is  $Y_i(t) = K_i(t)^\alpha [A(t)L_i(t)]^{1-\alpha}$ ,  $0 < \alpha < 1$ , and that  $A(t) = B \sum_i K_i(t)$ ,  $B > 0$ . Here  $K_i$  and  $L_i$  are the capital and labour used by firm  $i$ .  $K(t)$  is the aggregate capital stock, with initial value  $K(0) > 0$ . Labour is supplied inelastically and there is no population growth;  $\sum_i L_i(t) = L(t) = 1$ . Factor and product markets are perfectly competitive; capital and labour earn their *private* marginal products. There is no depreciation. The economy is populated by a representative household, which owns the capital stock. The lifetime utility of this household is given by:

$$U = \int_0^{\infty} e^{-\rho t} \ln(C(t)) dt, \quad \rho > 0, \quad (1)$$

where  $C(t)$  is the level of consumption and  $\rho$ , the rate of time-preference, is sufficiently high.

There is a Government in this economy. It taxes the capital income received by the household at some pre-determined rate  $\tau$ . It runs a balanced budget in all periods, and gives a lump-sum transfer of  $G(t)$  to the household.

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(a) (i) What are the private marginal products of capital and labour at firm  $i$ , as functions of  $K_i(t), L_i(t), K(t)$  and the parameters of the model?

*Answer [10 marks]*

(ii) Explain why the capital-labour ratio must be the same at all firms.

*Answer [5 marks]*

(iii) What are the wage,  $w(t)$ , and the gross interest rate,  $r(t)$ , as functions of  $K(t)$  and the parameters of the model.

*Answer [5 marks]*

(b) (i) Write down and describe the household's current-value Hamiltonian.

*Answer [5 marks]*

(ii) Write down the associated FOCs and transversality condition.

*Answer [10 marks]*

(iii) Using your answers so far, solve for the growth rates of consumption, output and capital in the decentralised equilibrium as functions of  $K(t)$  and the parameters of the model?

*Answer [25 marks]*

(c) Consider the role of Government in this economy. Assume its only objective is to maximise the utility of its citizens. It has two policy levers to achieve this. It must choose one or the other.

(1): Nationalise all production, whilst continuing to pay capital and labour their marginal products, and setting  $\tau = 0$ . *Hint: you can interpret nationalisation as resulting in a single large firm, with an aggregate production function.*

(2): Choose some level of  $\tau$ .

(i) Under option (1), what is the new growth rate of consumption and output in equilibrium? Discuss whether or not a benevolent social planner could do better than Government using this nationalisation option. *Hint: you do not need to solve the social planner's problem.*

*Answer [5 marks]*

**(ii)** Using your answers to (b)(iii) and (d)(i), what choice of  $\tau$  would result in options (1) and (2) giving the same growth outcomes? How does this depend on  $\alpha$  and/or  $B$ ? Discuss.

*Answer [20 marks]*

**(iii)** Comment on the reality or wider implications of this model and these results.

*Answer [15 marks]*

[Add discussion, a couple of paragraphs at least]