Macro 1: Extra Tutorials*

Problems

Latest version: 29/12/2016

Introduction

These problems are designed to encompass the basic mathematical requirements of Macro 1. In some cases the algebra and derivations may seem onerous. This is somewhat deliberate to give us plenty to practice. Read the *hints* carefully. In the tutorials we will slowly go through every step in the derivations and revise the relevant mathematical methods. e.g. How do you apply the chain/product rule? How do you manipulate exponents? Or even just, how did you get from there to there? You should always let me know if I am assuming too much/little knowledge. My hope is that after four or five sessions you will feel a little more confident attacking problems like these in the exams.

Unlike for your lectures and formal tutorials, the problems and models described here are not to be considered "course material." They have not necessarily been approved by the lecturers, and I will not be providing written solution guides.

Given I am volunteering extra time for these sessions, and they are informal / off-timetable, I really do hope you can come prepared. The best way to learn is to practice, not to just listen.

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Question 1 - Handling production functions and the Solow Model

- Consider a modified Solow type economy whereby output $Y(t)$ is produced by a representative firm using three factors of production: Capital $K(t)$, High-skilled labour $H(t)$ and Low-skilled labour $L(t)$;
  i.e. $Y(t) = F(K(t), A_L(t)L(t), A_H(t)H(t))$, where $A_H(t)$ and $A_L(t)$ are levels of labour augmenting technology which potentially differ.

- More specifically,

$$Y(t) = \left(\left(\beta K(t)^{\frac{\sigma-1}{\sigma}} + (1-\beta)(A_L(t)L(t))^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right)^{\alpha} (A_H(t)H(t))^{1-\alpha},$$

  where $\sigma > 1$, $\alpha \in (0,1)$ and $\beta \in (0,1)$.

- The population of High and Low-skilled labour are constant; $H(t) = H$, $L(t) = L$. Also, define the total workforce as $N(t) = N = H + L$

- Technology evolves exogenously according to $A_H(t) = A_H(0)e^{g_H t}$ and $A_L(t) = A_L(0)e^{g_L t}$

- Consumers save a constant fraction s of their income; i.e. $C(t) = (1-s)Y(t)$ and $S(t) = sY(t)$. There is perfect competition and capital evolves according to

$$\dot{K}(t) = I(t) - \delta K(t) \quad \delta \in (0,1),$$

  where investment $I(t) = Y(t) - C(t)$.

- The price of output is normalised to one. Let $R(t)$ denote the rental price of capital and $W_H(t)$ and $W_L(t)$ the wage rates.

(a) Discuss whether $F(K(t), A_L(t)L(t), A_H(t)H(t))$ has any properties of a neoclassical production function. Include brief discussion of the economic meaning for each of these properties with regards this production technology and each factor.

[Hint: you are expected to know the set of properties of a neoclassical production function. Check the lecture notes, Romer, or google if you have forgotten. The parameter restriction $\sigma > 1$ should simplify your answer a lot. Begin by checking if the production function exhibits Constant Returns to Scale, and then use this

answer to simplify the algebra, as I demonstrated in Tutorial 2. You need only consider cases for H and either L or K. The solutions to Tutorial 2, Question 1 may also help here.]

(b) What parameter restrictions would return us to the standard Solow model Cobb-Douglas production function from your lectures, interpreting then $L(t)$ as just generic labour?

(c) Characterise the equilibrium of the economy described above for any $t \geq 0$ given initial values $K(0) = K_0 > 0$, $A_L(0) = A_{L,0}$ and $A_H(0) = A_{H,0}$.

[Hint: There is no consumer problem in this model set-up. You should also be able to use your answer from (a) and information given above to write-down the FOCs of the firm's problem; you do not need to set-up the firm's maximisation problem.]

(d) Rewrite both the production function and the equilibrium equation for the evolution of capital, that you find in (c), in per worker terms. Denote lower case terms as per worker. i.e. $y(t) = Y(t)/N = f()$.

(e) Simplify the model by letting $A_L(t) = A_H(t) = A(t)$, $\forall t$, and $g_H = g_L = g$. Find an **implicit** expression for the balanced growth path value of capital per effective worker $\tilde{k}^*$ (you can assume the parameter values are such that it actually exists).

(f) Discuss how $\tilde{k}^*$, $\tilde{y}^*$ and $(W_H(t)/W_L(t))$ at the balanced growth path depend on h (the share of the workforce who are high-skilled).

[Hint: You might find this part algebraically/analytically challenging. You can attempt to crunch through the algebra, but a neater answer might use some economic intuition and/or a graphical analysis.]

(g) Find $\left(\dot{\frac{W_H(t)}{W_L(t)}}\right) / \left(\frac{W_H(t)}{W_L(t)}\right)$ at the balanced growth path in terms of model parameters only. Would you say that technological change in this simplified version of the model is skill-biased?

[Extra credit / something to think about...]

(h) In light of your answers to (f) and (g), would the production function, parameter restrictions and simplifications here be your first choices, if you were writing down a model to explain and test why the skill-premium in developed countries has significantly increased in recent decades? What changes would you make?

Question 2 - Neoclassical Growth Model (I)

Consider a continuous time Neoclassical / Ramsey-Cass-Koopmans model which is in almost all respects identical to the one you were introduced to in lectures, with $\rho - n > (1-\varepsilon)g$. The only difference is that now there is a government which can tax investment income at rate τ and capital does not depreciate. Any revenue, $G(t)$, the government collects from the tax, is returned to households in the form of lump sum transfers.

(a) Set up the representative household's maximisation problem, write-down the associated current value Hamiltonian and find all of the first order conditions.

(b) Using your results from (b), derive the laws of motion for consumption and capital per effective worker. Discuss why for any initial value of $K(0) > 0$ the equilibrium path is unique and the economy will converge to some balanced growth path $(\tilde{k}^*, \tilde{c}^*)$.

(c) Imagine that $\tau = 0$ before some initial period t_0. Assume that the economy was at the BGP before t_0. Using the phase diagram for the economy, discuss the impacts on the economy thereafter of an unanticipated increase in the tax rate to $\tau > 0$ at t_0.

(d) Repeat the analysis in (c), but now imagine that the tax rate will be increased at some $t_1 > t_0$, but this is announced to households and firms at t_0.

(e) In this model, would households have preferred to be warned that the tax was coming?

[Hint: You should only need to appeal to the results of your phase diagram analysis and the general features of the Neoclassical Growth Model to answer this fully.]

(f) Write-down the equilibrium path of capital per effective worker if government did not return the collected tax to households, but instead made purchases which do not affect utility; i.e. they do not add to the capital stock. How would your answers to (c) and (d) change?

[Hint: "Write-down" implies that you do not need to repeat parts (a)-(b). It should be straightforward to see how the laws of motion would change.]

(g) In light of your answers here, would households be better off (in terms of welfare), relative to before t_0, if government introduced a subsidy $\tau < 0$, which was funded by lump-sum taxes on households?

[Hint: This is a discussion question. You should not require any further algebraic manipulation, though diagrams could be used to illustrate your answer.]

Question 3 - Neoclassical Growth Model (II)

Attempt all parts (a-e) of Question 2 (Section A) from the 2015/16 Macro 1 Mock Exam [available on Learn, with Jan's solution guide].

Hint: For parts (a) (and (d)), can you simplify the household's (or planner's) problem before setting up the Current Value Hamiltonian, at least relative to the solution given in the answer guide?

(f) Suppose in the decentralised economy the government can impose a subsidy at rate τ on the household's gross remuneration from research capital. The subsidy is funded by a lump-sum tax, given by $T(t) = \tau Q(t) B_x(t)$. In terms of model parameters only, what value of τ should the government choose to achieve the social planner's equilibrium in the decentralised economy? Discuss how this value varies with β.

Question 4 - Consumption Theory (I)

(i) Attempt all parts (a-c) of Section B: Question 1. from the May 2016 Macroeconomics 1 Exam. [Past exam papers available online via the Uni. Library]

Hint: For part (c), assume that $\varepsilon_t \overset{i.i.d.}{\sim} N(0, \sigma^2)$

(ii) Attempt all parts of Problem 8.14. from Romer 4th ed.

Question 5 - Consumption Theory (II) & Unemployment

From the May 2016 Macroeconomics 1 Exam. [Past exam papers available online via the Uni. Library]:

(i) Attempt all parts (a-c) of Section B: Question 2.

Hint: Try to solve this problem more generally first, assuming that $\beta R \neq 1$*, and then only at the "last moment" assume* $\beta R = 1$*. Also, note that the variance of the uniform distribution is equal to* $(b-a)^2/12$*, where b and a are it's limits. Or more simply, note that the variance of* Y_1 *is just increasing in* $(b-a)$*.*

(ii) Attempt all parts (a-c) of Section B: Question 3.

Hint: For the comparative statics you should describe using figures and words (economic meaning) the new equiibrium path of the endogenous variables. I.e. not just the new steady states.