Economics of Sport (ECNM 10068)

Lecture 9: Game Theory in the Arena

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Game Theory in the Arena

Issues covered:

Two applications of Game Theory to strategic situations in sports:

1. Which direction to serve in a tennis match?
2. Which way to shoot and which way to dive in penalty kicks?

Main reading:

Chapter 5, Dobson-Goddard "The Economics of Football" 2nd ed. Cambridge 2011;

Walker, M., and J. Wooders. 2001. "Minimax Play at Wimbledon." American Economic Review, 91 (5): 1521-1538;

Chiappori, P.-A., S. Levitt, and T. Groseclose. 2002. "Testing Mixed-Strategy Equilibria When Players Are Heterogeneous: The Case of Penalty Kicks in Soccer." American Economic Review, 92 (4): 1138-1151.

Games, Strategy & Unpredictability

- In strategic situations, being unpredictable to one's opponent(s) is important.
- The origins of modern game theory came from trying to understand these situations.
- Mixed-strategy play is key to the theoretical description of how games which require unpredictability should be (are) played.
- For example:
  - Minimax theorem – von Neumann (1928)
  - Nash equilibrium in mixed strategies –
    von Neumann & Morgentsen (1944); Nash (1951)

Testing the theory of strategic play & unpredictability

- In experimental settings or labs, the predictions of mixed-strategy play in strategic games have not fared well.
- Players in such settings consistently do not play to the theory.

  Why?

  Even simple games are hard to play well, especially when doing so requires unpredictability – e.g. card games.
- But, professional sports could provide simple strategic situations where the participants are experts on how to play the games.
- However, most sporting situations are complex - many choices, multiple players, different contexts etc.

Serve and Return in Tennis - Walker & Wooders (2001; *AER*)

- A testable example from championship professional tennis:
- The serve is an important factor in who wins points in Tennis.
- Almost all (first) serves are (aimed) delivered as far to the left or right of the service court as possible (assumption, can be relaxed to model other options, such as spin on the ball).
- Each point has two outcomes: server wins or loses.
- The server's action is observable.
- There is plenty of data on repeated strategic interactions between the same two players.

A model of the Serve in Tennis

- Each point is a 2 *X* 2 normal-form game, between two players.

		RETURN (pre-meditate)	
		LEFT	RIGHT
SERVE	LEFT	$\pi_{LL}, 1-\pi_{LL}$	$\pi_{LR}, 1-\pi_{LR}$
	RIGHT	$\pi_{RL}, 1-\pi_{RL}$	$\pi_{RR}, 1-\pi_{RR}$

Source: lecturer, also reflected in Walker & Wooders (2001; *AER*).

A model of the serve in Tennis

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SERVE	LEFT	π_{LL}	π_{LR}
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Source: lecturer, also reflected in Walker & Wooders (2001; *AER*).

A model of the Serve in Tennis (continued)

- Each point is a 2 *X* 2 normal-form game, between two players.
- The Server (s) plays Left (L) or Right (R).
- Simultaneously, the Returner (r) guesses Left (L) or Right (R): i.e. they pre-meditate, or overplay one way or the other.
- The Game is a reduced form of a tennis point - does not attempt to model what happens after the (first) serve is delivered.
- The expected payoffs to the server are given by π_{sr}, equivalent to the probability of winning the point after the players have made their choices; i.e. the payoff to the returner is $1 - \pi_{sr}$.
- In a dynamic "match" version of the model, it can be shown that players should optimally play each point like it is the only one.

Assumptions of the model:

- $\pi_{LL} < \pi_{RL}$ and $\pi_{RR} < \pi_{LR}$: The server is more likely to win if they serve away from the direction the returner pre-meditates.

- This is equivalent to:

"Assumption 1: Every point in a tennis match is played as a 2 *X* 2 *constant-sum normal-form game with a unique equilibrium in strictly mixed strategies."* (Walker & Wooders, 2001)

How to find the equilibrium: The Minimax theorem

Some definitions:

- Maximin value, m_i: the highest expected value player i can assure themselves
- Maximin strategy $\widetilde{\mathbf{p}}_i$: a (mixing) strategy that assures player i of his Maximin value.
- Minimax value, M_i: the lowest expected value player i's opponent can limit them to.
- Minimax strategy, $\mathbf{p}_i$: a (mixing) strategy which limits player i's opponent to his Minimax value.
- Constant-sum game: the sum of the players' payoffs is the same for every profile of strategies, i.e. in each cell, the payoffs sum to a constant K.
  In the Serve & Return Tennis game, $K = 1$.
  If $K = 0$, then it is known as a "zero-sum game."

- The Minimax theorem:
  Every finite constant-sum two-player game has optimal mixed strategies, where $m_i = M_i$ for either player.

- In other words, whatever maximum expected value a player can assure for themselves, is also the minimum that their opponent can limit them to.

- When there is more than one optimal mixed strategy, there are infinitely many.

- This theorem of von Neumann is often referred to as the beginning of Game theory, only later to be generalised considerably by Nash.

- The Minimax theorem continued:

  For each player i, with opponent $-i$, the minimax and maximin values are given by:

$$M_i = \min_{\mathbf{p}_{-i}} \max_{\mathbf{p}_i(\mathbf{p}_{-i})} E\left[\Pi(\mathbf{p}_i, \mathbf{p}_{-i})\right] = \max_{\mathbf{p}_i} \min_{\mathbf{p}_{-i}(\mathbf{p}_i)} E\left[\Pi(\mathbf{p}_i, \mathbf{p}_{-i})\right] = m_i \ ,$$

  where $E\left[\Pi(\mathbf{p}_i, \mathbf{p}_{-i})\right]$ is the expected payoff function.

Applying the Minimax theorem to the Serve in Tennis game: an example

		RETURN (pre-meditate) LEFT	RETURN (pre-meditate) RIGHT
SERVE	LEFT	0.58	0.79
SERVE	RIGHT	0.73	0.49

Source: lecturer, also reflected in Walker & Wooders (2001; *AER*).

Applying the Minimax theorem to the Serve in Tennis game: an example

Let p denote the probability of playing Left.

$$M_s = \min_{p_r} \max_{p_s(p_r)} E\left[\Pi(p_s, p_r)\right]$$

$$M_s = \min_{\mathbf{p}_r} \max_{\mathbf{p}_s(\mathbf{p}_r)} p_s\left[p_r 0.58 + (1-p_r)0.79\right] + (1-p_s)\left[p_r 0.73 + (1-p_r)0.49\right]$$

$$M_s = \min_{\mathbf{p}_r} \max_{\mathbf{p}_s(\mathbf{p}_r)} 0.49 + 0.24p_r + p_s(0.3 - 0.45p_r).$$

$$M_s = \min_{\mathbf{p}_r} \begin{bmatrix} 0.79 - 0.21p_r & \textit{if } p_r \leq 2/3 \\ 0.49 + 0.24p_r & \textit{if } p_r \geq 2/3 \end{bmatrix} = 0.65$$

with $p_r^* = 2/3$. So, the server when using a minimax/maximin strategy wins 65% of the time in this game.

Now for practice, show that this is equivalent to the server's maximin value. Also, solve explicitly the receiver's minimax problem, and show that $M_r = 0.35$ *and* $p_s^* = 1/3$

The Nash Equilibrium, minimax & Best Responses

- The minimax strategy is ‘conservative.’
- If the opponent is not playing his own minimax strategy, there could be a way to respond, and keep him below that value.
- Best response function – a player’s optimal strategy for any given strategy of their opponent:

$$\widehat{\mathbf{p}}_i(\mathbf{p}_{-i}) \in \arg\max_{p_i} E\left[\Pi(\mathbf{p}_i, \mathbf{p}_{-i})\right]$$

- Nash Equilibrium: a profile of strategies which are mutual best responses.
- **In any finite, two-player and constant-sum game, it is a Nash Equilibrium for the players to both play their minimax strategies.**
  You do not need to be able to prove this, but you should be able to check this is the case, using the example Serve in Tennis game.

Testable predictions of the Serve in Tennis game

If players are following their optimal mixing minimax strategies:

- Observed choices in any match, will be independent draws from a binomial process, depending on which player is serving (and potentially whether or not they are serving to the deuce- or ad-court side.)

- Players should have the same probability of winning the point, whichever direction they serve.

Do professional tennis players optimally mix strategies?

Walker and Wooders (2001) find that among the top male players at major championships:

- The hypothesis that players use optimal minimax strategies cannot be rejected.

- But, the serial independence assumption (each point is a new game, with no memory, history, dependence on past choices/outcomes) is not satisfied; tennis players switch from one action to another too frequently.

- Optimal mixed strategy behaviour may be rejected in the lab, but there is evidence that 'game experts' in sports, who perhaps more fully understand the rules and the payoffs, do mix optimally.

Penalty kicks in football - Chiappori et al. (2002; *AER*)

Some preliminaries:

- The max. speed a penalty taker kicks the football is 125 mph.
- At this speed, the ball enters the goal two-tenths of a second after it is kicked: a keeper who jumps after the ball is kicked cannot possibly save the shot (unless it is aimed at him).
- Generally, both kicker and goalkeeper must decide which way to go beforehand.
- Approx. 4/5 of penalties are scored. Approx 50% of games are tied or end with a one goal difference – the outcome of a penalty kick matters.
- In reality, each player has a 'natural' side to kick to (the left if right-footed) – assume for the model all players are right-footed.

Penalty kicks in football – the model

- Each penalty is a 3 *X* 3 normal-form game, between two players.

Penalty taker (shoots)	Goalkeeper (dives): LEFT	CENTRE	RIGHT
LEFT	P_L	π_L	π_L
CENTRE	μ	0	μ
RIGHT	π_R	π_R	P_R

Source: lecturer, also reflected in Chiappori et al. (2002; *AER*).

Penalty kicks in football – the model (continued)

- Each penalty is a 3 *X* 3 normal-form game, between two players.
- The kicker shoots Left (*L*), Centre (*C*) or Right (*R*).
- Simultaneously, the keeper dives Left (*L*) or Right (*R*), or does not dive (*C*).
- The cells on the previous slide give the expected payoffs to the kicker, which are equivalent to the probability of scoring. The model is assumed to be a zero-sum game; i.e. the payoff to the keeper is minus one times that of the kicker in each cell.

Assumptions of the model:

ASSUMPTION SC ("Sides and Centre"):
$\pi_R > P_L$; $\pi_R > \mu$; $\pi_L > P_R$; $\pi_L > \mu$,
i.e. probability of scoring is greater if the keeper goes the 'wrong' way.

ASSUMPTION NS ("Natural side"):
$\pi_L \geq \pi_R$; $P_L \geq P_R$
i.e. probability of scoring is greater if the kicker shoots to his 'natural' side.

ASSUMPTION KS ("Kicker's side"):
$\pi_r - P_r \geq \pi_L - P_L$
i.e. kicks to the natural side are harder to save.

Unique mixed strategy equilibrium

There exists a unique mixed strategy Nash Equilibrium: If

$$\mu \leq \frac{\pi_L \pi_R - P_L P_R}{\pi_R + \pi_L - P_L - P_R} \tag{1}$$

then both players randomize over $\{L, R\}$ only. Otherwise, both players randomize over $\{L, C, R\}$.

Properties of the equilibrium

1. The kicker's and keeper's randomisation are independent.
2. The scoring probability is the same irrespective of the direction the ball is actually kicked, or irrespective of the direction the keeper dives (including the centre whenever the ball is kicked / keeper stays there with positive probability).
   1. and 2. are standard properties of this class of mixed strategy equilibria, like in the Tennis game
3. The kicker is more likely to choose centre than the keeper [SC].
4. The kicker chooses his natural side less often than the keeper [SC].
5. The keeper always chooses the kicker's natural side more often than his 'nonnatural' side [SC & NS].
6. The kicker chooses his natural side more often than his 'nonnatural' side [SC & KS].
7. (L,L) is more likely than (L,R) and (R,L), which in turn are both more likely than (R,R) [SC, NS & KS].

Properties of the equilibrium

- In reality, it is rare that there are multiple penalties with the same kicker and keeper in the same match.
- In testing the theory of optimal mixed strategy behaviour, after aggregating over matches and allowing heterogeneity, there are likely to be econometric issues which need to be addressed — e.g. selection bias.
- Exogenous variables could matter too – e.g. the time of penalty in the match.
- Nonetheless, some of the predictions are robust to aggregation over matches and can be taken directly to the data (3 - 7).

- Chiappori et al (2002) also address the econometric issues from aggregation and find empirically that:
- they cannot reject the assumption that kicker and keeper play simultaneously – the direction played does not appear to influence the opponent's choice of action;
- kickers play as if all keepers are identical;
- but, the strategy chosen by a keeper does depend on a kicker's past history;
- all theoretical predictions which are robust to aggregation are satisfied (3 - 7);
- they cannot reject (conditionally) that scoring probabilities for kickers are equal across right, left and centre;
- they cannot reject equal scoring probabilities with respect to goalies jumping left or right (low powered test: keepers almost never stay in the centre).

A practice exam-type problem

S and *V* play each other at tennis a lot. Their matches can be modelled as a series of point games, as per Walker and Woods (2001; AER). The outcomes of each point depend on the direction that either player simultaneously decides to serve or pre-meditate their return of serve: **L**eft or **R**ight. On the next slide, historical data (fictional) is presented on the outcomes of *S* and *V*'s point games against each other. For example, when *S* serves, and both players go Left, then she has in the past won the point 58% of the time. When *V* serves, and both players go Left, then she has in the past won 65% of the time. Assume that both players have perfect knowledge about these historical head-to-head records, and they perfectly reflect their respective and relative abilities. Assume that all points are independent of one another, and both players always play to win.

Practice exam Q - A Serve & Return game of a Tennis point, depending on who serves

		RETURN (pre-meditate)	
		LEFT	RIGHT
SERVE	LEFT	0.65 V / S 0.58	0.7 V / S 0.79
	RIGHT	0.65 V / S 0.73	0.54 V / S 0.49

Source: lecturer, inspired by Walker and Woods (2001; AER).

Continued:

(i) If both players use optimal minimax strategies, then what percentage of points do *S* and *V* win on their respective serves.

(ii) Without any derivation, explain in words why your answers to (i) must also be the unique mixed-strategy Nash equilibria of the tennis point games between Serena and Venus.

(iv) Describe how you would test empirically whether or not *S* and *V* in reality use optimal minimax strategies, as well as the assumption that all points they play together are independent of one another.

Outline answer:

(i) In general, we can write the server's minimax value as:

$$M_s = \min_{p_r} \max_{p_s(p_r)} E\left[\Pi(p_s, p_r)\right] \ ,$$

where s and r denote server and returner, respectively. Let payoffs $\pi_{sr} \in \{\pi_{LL}, \pi_{LR}, \pi_{RR}, \pi_{RL}\}$ be the probabilities of success in each game cell for the server. Then we can show that in general for this type of game, with assumptions as per Walker and Wooders (2001; *AER*):

$$M_s = \pi_{RR} + \frac{(\pi_{RL} - \pi_{RR})(\pi_{LR} - \pi_{RR})}{\pi_{RL} + \pi_{LR} - \pi_{LL} - \pi_{RR}} \ .$$

Substituting in the values given by the question, we find that $M_s^S = 0.65$ (as per earlier in the lecture) and $M_s^V = 0.65$. The two players are evenly matched, despite S having the stronger serve, and V the stronger return. They both win 65% of their points on serve against the other.

(ii) ...

(iii) Please do the required reading for this lecture - you might even enjoy it!