Economics of Sport (ECNM 10068)

Lecture 8: Theories of a League - Part II

Carl Singleton[1]

The University of Edinburgh

13th March 2018

[1] Carl.Singleton@ed.ac.uk

Theories of a League - Part II

Issues covered:

- Continued: a theoretical model of resource allocation in sports leagues (a.k.a. the labour market for talent)

Main reading:

Chapter 2, Dobson-Goddard “The Economics of Football” 2nd ed. Cambridge 2011; Noll, R. (2003). Sakovics, J. & Burguet, R. (2018). “Bidding for talent in sport”, ESE Discussion Paper, 285.

A recap of Lecture 7: Competition for talent/resources when the pool is fixed

The 2-team fixed talent pool model ($n = 2$)

- There are 2 teams, denoted by $i \in \{1, 2\}$.
- They hire talent t_i, where the pool of talent is **fixed** (closed), i.e. $t_1 + t_2 = T = 1$ (normalisation).
- Team i's revenue depends on three factors:
    1. Market size: m_i
    2. The overall quality of league standards: $\bar{t} = T/n = 1/n$
    3. The relative quality of team i: $\tau_i = t_i/\bar{t} = nt_i$
- The equilibrium market price/marginal/average cost of talent is constant, c (endogenous - the price clears the market for talent).
- The talent choice of team i affects the quantity hired by the other teams.

- Revenue functions are given by:

$$R_i = \frac{\eta m_i(2-\theta)(t_i - 2\xi t_i^2)}{2} = \psi m_i(t_i - \phi t_i^2) \ , \tag{1}$$

where $\psi = \frac{\eta(2-\theta)}{2}$ and $\phi = 2\xi$
(see last Lecture and textbook on derivation from n team model).

The equilibrium depends on the objectives of the teams:

- Profit maximisation: choose talent up to the point where marginal revenue = marginal cost; $MR_i = MC_i = c$.
or
- Win-percent maximisation, with zero-profit constraint: choose talent up to the point where average revenue = average cost; $AR_i = AC_i = c$.

Profit maximisation equilibrium: 2-team fixed talent pool model

- The equilibrium values of $\{t_1^\pi, t_2^\pi, c^\pi\}$ are determined by three equations:

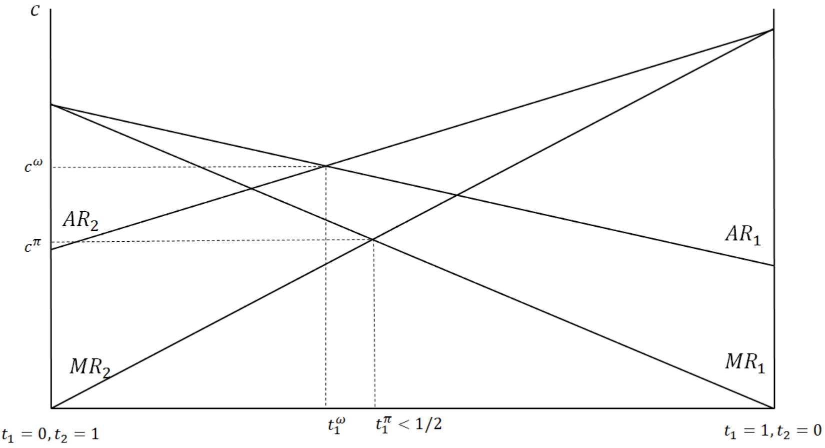
$$
\begin{aligned}
MR_1 =& \psi m_1(1-2\phi t_1^\pi) = c^\pi \ , \qquad (2)\\
MR_2 =& \psi m_2(1-2\phi t_2^\pi) = c^\pi \ , \qquad (3)\\
1 =& t_1^\pi + t_2^\pi \ , \qquad (4)
\end{aligned}
$$

with solution:

$$
\begin{aligned}
c^\pi =& \frac{\psi m_1 m_2 (2-2\phi)}{m_1+m_2} \ , \qquad (5)\\
\frac{t_1^\pi}{t_2^\pi} =& \frac{m_1 - m_2 + 2\phi m_2}{m_2 - m_1 + 2\phi m_1} \ . \qquad (6)
\end{aligned}
$$

- We can represent this graphically as follows:

Equilibrium - 2-team fixed talent pool - $m_2 > m_1$



Source: lecturer

Equilibrium - 2-team fixed talent pool - continued

- As drawn, $m_2 > m_1$, therefore $t_2^* > t_1^*$.
- The relative difference in talent would be larger if the teams were both win-percent rather than profit maximisers: $\frac{t_1^\pi}{t_2^\pi} > \frac{t_1^\omega}{t_2^\omega}$, which means that competitive balance is also reduced.
- The total/marginal/average cost of talent (player salaries / market price), c^*, is greater if teams are win-percent maximisers.

Other features of the equilibrium in this type of model:

- Driving this eq. is the fact that

$$\frac{\partial(t_i + t_j)}{\partial t_i} = 0 \quad \Rightarrow \quad \frac{\partial t_j}{\partial t_i} = -1 \; , \tag{7}$$

  i.e. the pool is fixed, and all talent is hired, so any additional talent hired by team *i* just reduces team *j*'s quantity of talent by the same amount.
- The eq. adheres to the 'Invariance Principle': it is independent from the initial allocation of talent across the teams in the leagues.
- Therefore, the eq. is 'Coasian' (Coase's theorem), and extends the First Welfare Theorem.
- This is because the model assumes property rights and efficient/cashless negotiation: there are no transaction costs.

Extensions of the model:

- Sports league and market institutions:
  - Rules on player salaries, e.g. a total payroll cap
  - Revenue sharing
- What does an Open market for talent look like?
- Can we unify the pecuniary (profit) and non-pecuniary (e.g. win-percent) motives of team owners in one objective function, and still use the first order principle?
- Can we micro-found a market where only some of the resources are contested, endogenously?

Payroll cap - the average of team revenues case

- We extend the 2-team fixed model described before, where we assume $m_2 > m_1$.

- Assume teams are initially profit maximisers, where in eq. $t_2^{\pi} > t_1^{\pi}$, and the total salary cost for team i is $(c^{\pi} t_i^{\pi})$.

- We consider a payroll cap which is intended to reduce competitive inequality.

- An example - NFL salary caps: link

- The level of the cap is set as a fraction of the average revenues in the league, i.e. the league exogenously sets $\kappa > 0$:

$$CAP(t_1, t_2) = \kappa(R_1 + R_2)/2 \geq ct_i \ . \tag{8}$$

What is the equilibrium with a payroll cap?

- Solution method: first check whether the level of talent chosen by team 2 in the profit max. equilibrium without a cap would be constrained for some level of κ:

  If

  $$c^{\pi} t_2^{\pi} \leq \kappa(R_1^{\pi} + R_2^{\pi})/2 \,, \tag{9}$$

  then neither team will be constrained. The cap is irrelevant.

  If

  $$c^{\pi} t_2^{\pi} > \kappa(R_1^{\pi} + R_2^{\pi})/2 \,, \tag{10}$$

  then at least team 2 will be constrained by the payroll cap.

- Assuming team 2 is only affected by the payroll cap, but team 1 is not. Then the new equilibrium will be given by the solution of $\{\tilde{t}_1^\pi, \tilde{t}_2^\pi, \tilde{c}^\pi\}$:

$$\widetilde{MR}_1^\pi = \psi m_1(1 - 2\phi \tilde{t}_1^\pi) = \tilde{c}^\pi \ , \tag{11}$$
$$\tilde{c}^\pi \tilde{t}_2^\pi = \kappa(\tilde{R}_1^\pi + \tilde{R}_2^\pi)/2 \tag{12}$$
$$1 = \tilde{t}_1^\pi + \tilde{t}_2^\pi \ , \tag{13}$$

where team 2 hires up to the cap.

- But then, we need to check whether team 1 is also constrained: If

$$\tilde{c}^\pi \tilde{t}_1^\pi \leq \kappa(\tilde{R}_1^\pi + \tilde{R}_2^\pi)/2 \ , \tag{14}$$

then what we described above is the equilibrium for given κ. If

$$\tilde{c}^\pi \tilde{t}_1^\pi > \kappa(\tilde{R}_1^\pi + \tilde{R}_2^\pi)/2 \ , \tag{15}$$

then both teams are constrained by the cap.

- In this case, the equilibrium is given by $\{\hat{t}_1^{\pi}, \hat{t}_2^{\pi}, \hat{c}^{\pi}\}$:

$$\hat{c}^{\pi}\hat{t}_1^{\pi} = \kappa(\hat{R}_1^{\pi} + \hat{R}_2^{\pi})/2 \tag{16}$$

$$\hat{c}^{\pi}\hat{t}_2^{\pi} = \kappa(\hat{R}_1^{\pi} + \hat{R}_2^{\pi})/2 \tag{17}$$

$$1 = \hat{t}_1^{\pi} + \hat{t}_2^{\pi}\ , \tag{18}$$

where both teams hire up to the cap.

- If only team 2 is affected by the cap, then competitive balance is improved: $\tilde{t}_2^{\pi} < t_2^{\pi}$. Intuitively also, $\tilde{c}^{\pi} < c^{\pi}$.

- If both teams are affected by the cap, then $\hat{t}_1^{\pi} = \hat{t}_2^{\pi} = 1/2$, and $\hat{c}^{\pi} < \tilde{c}^{\pi}$.

Payroll cap - the own revenue to costs ratio case

- An alternative payroll cap could be based on each team in the league spending no more than a percentage ρ of their revenues on player salaries.

- This type of cap is widely in place in European football - note, the cap could be greater than 100%, and in reality, the interpretation of 'Revenues' is somewhat loose / manipulated.

- An example - UEFA FFP - the case of PSG/Man City: link

- In the model, this cap can be written as:

$$CAP(t_i) = \rho R_i \geq c t_1 \ . \tag{19}$$

- The solution method, for finding the equilibrium with such a cap in place, is the same as described above with the first type of payroll cap.

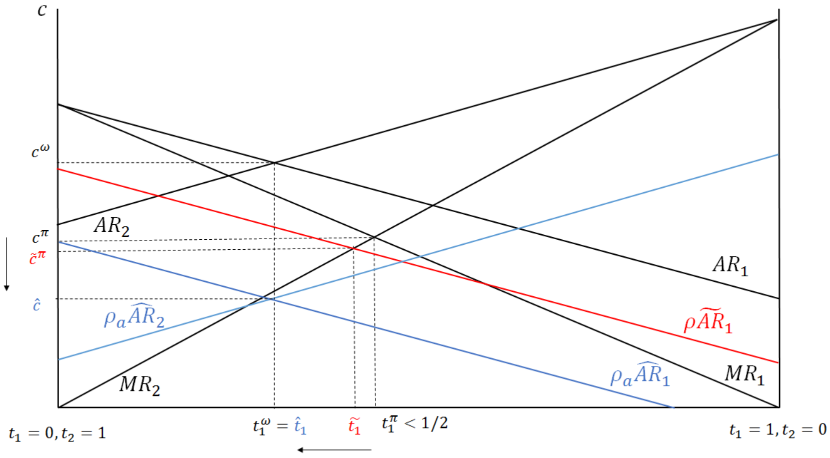
- But, with the functional form of the team objective function from before, it is now team 1 who is more likely to be affected by the cap.

- This is clear, because we can re-write the cap as: $R_i/t_i = AR_i = c/\rho$

- Team 1's average revenue in equilibrium will always be strictly below team 2's, when $m_2 > m_1$.

- We can represent the effects of such a cap on the equilibrium graphically:

Equilibrium - 2-team fixed talent pool - $m_2 > m_1$
The effects of a payroll cap based on the ratio of total revenue and talent costs.



Source: lecturer

- This type of cap can only decrease competitive balance. In the model, the poorer teams are affected first.

- If ρ is very low, and both teams are affected, then the model reverts to the win-percent maximisation equilibrium for the allocation of talent.

- However, whereas a win-at-all-costs mentality of teams tends to increase player salaries, this type of payroll cap will decrease them, relative to unconstrained profit maximisation.

- So there is a trade-off w.r.t. the size of ρ. Optimally, it depends on how much the administration of the league wishes to control costs and player salaries, vs. how much competitive inequality they are willing to allow.

Revenue sharing

- We extend the 2-team fixed model described before, where we continue to assume $m_2 > m_1$.

- Assume teams are initially profit maximisers, where in eq. $t_2^\pi > t_1^\pi$.

- We consider a revenue sharing institution:

- Each team can keep a share α of their own revenues. A share $1-\alpha$ is collected by the league.

- Both teams receive half of the pooled revenues, collected by the league, i.e. a team's net revenue is given by

$$NR_i = \alpha R_i + (1-\alpha)(R_1 + R_2)/2 \; . \qquad (20)$$

Profit maximisation equilibrium: 2-team fixed talent pool model
Revenue sharing:

- The equilibrium values of $\{t_1^{share}, t_2^{share}, c^{share}\}$ are determined by three equations:

$$\frac{NR_1}{\partial t_1} = NMR_1 = c^{share} , \tag{21}$$

$$\frac{NR_2}{\partial t_2} = NMR_1 = c^{share} , \tag{22}$$

$$1 = t_1^{share} + t_2^{share} , \tag{23}$$

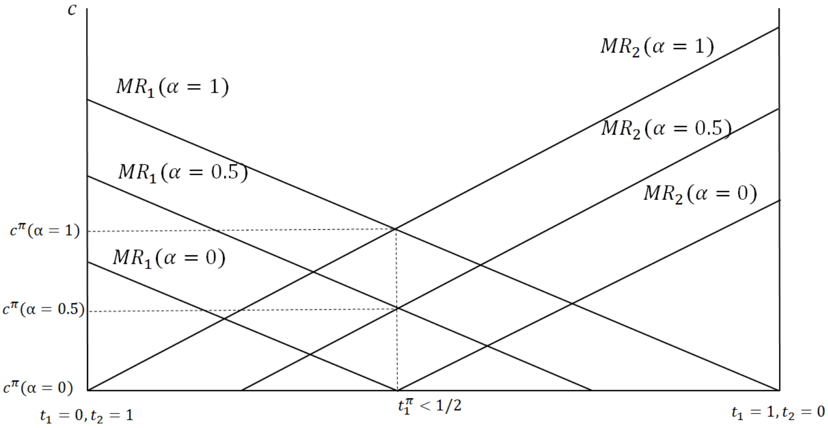
with solution:

$$c^{share} = \alpha \frac{\psi m_1 m_2 (2 - 2\phi)}{m_1 + m_2} = \alpha c^{\pi} , \tag{24}$$

$$\frac{t_1^{share}}{t_2^{share}} = \frac{t_1^{\pi}}{t_2^{\pi}} . \tag{25}$$

- We can represent this graphically as follows:

Equilibrium - 2-team fixed talent pool - $m_2 > m_1$
The effects of sharing $1 - \alpha$ of the team revenues.



Source: lecturer

- Revenue sharing in this model does not affect the equilibrium allocation of talents.
- The model is Coasian: whether or not individual teams or the league choose the profit maximising allocation of talent, the result is unchanged.
- However, individual action by teams in effect gives the talent bargaining power.
- If teams can agree to share revenues, then they can limit this bargaining power, increasing their collective and individual profits.
- If teams are win-percent maximisers, this result does not necessarily hold: revenue sharing would decrease competitive inequality.

An Open Model of Resource Allocation

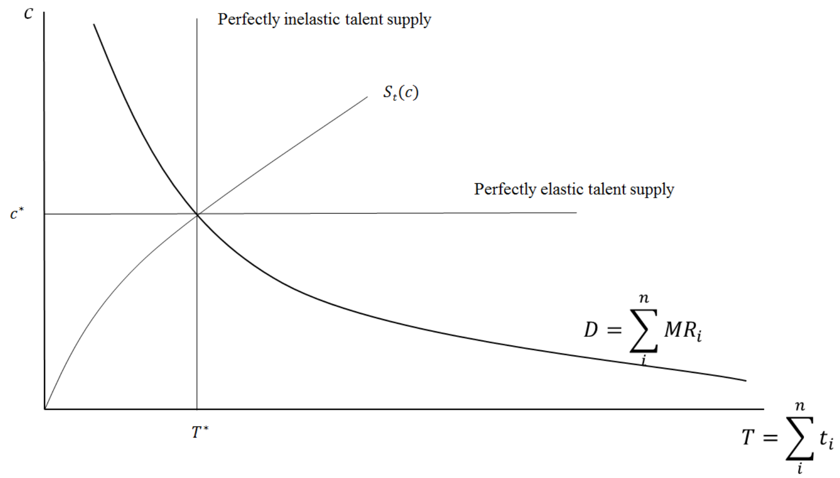
- In a fully open model:

$$\frac{\partial(\sum_j^n t_j)}{\partial t_i} = 1 \quad \Rightarrow \quad \frac{\partial(\sum_{j \neq i}^{n-1} t_j)}{\partial t_i} = 0\,, \tag{26}$$

  i.e. a team's own talent demand does not affect the amount that the other can hire.

- A completely Open model does not reflect any of the realities of the market for talent in sports.
- In this case, teams in the league would independently maximise their revenues, taking as given some market price c.
- An equilibrium would be found from summing each MR_i, to find the total market demand, and by then finding the intersection with some supply curve for talent $S_t(c)$, giving a market price c^*:

Equilibrium - n-team Open talent pool



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A more realistic micro-founded model of the labour market in a sports league: Burguet & Sakovics (B&S; 2018)

- There are two important features of a sports labour market missing from the fixed talent pool model.

  1. Competing objectives: in reality, a team can trade off profits and winning, or other non-pecuniary objectives.

  2. Teams contest some talent, whereas some talent goes uncontested.

- B&S proposed two extensions of the traditional framework (see also Fort (2015; *Scot. J. Pol. Econ.*) for the first extension).

1. Competing objectives

- The owners of sports teams often have alternative uses for their money.
- Benefactors (or sovereign wealth funds): instead of investing in the playing talent, they could purchase a yacht (or invest elsewhere).
- Supporter-owned teams: fans have other uses for their money
- Members-owned clubs, with multiple teams in different sports (e.g. FC Barcelona, Bayern Munich): can use revenues from one team to subsidise others.
- In the last lecture's practice problem, we modelled the effect of a Benefactor by relaxing the zero-profit constraint for a win-maximising team.
- But win-percent maximisation does not match the first order principle, so it is unsatisfying: e.g. Manchester City still runs itself day-to-day like a typical firm, but occasionally deviates to make a significant player purchase.
- The objective function of B&S generalises to cover the real-world examples of competing owner objectives.

- The traditional profit-max. way to model a sports talent labour market presents team objectives as

$$\max_t \left[B + R(t) - C(t)\right] \ , \tag{27}$$

where B is some cash endowment, $R(t)$ is revenues, and $C(t)$ is the cost function for talent.

- The traditional utility-max. way to model a sports talent labour market presents team objectives as

$$\max_t U(t) \quad s.t. \quad B + R(t) - C(t) \geq 0 \ , \tag{28}$$

where $U(t)$ is some strictly increasing utility function (could be based on win-percent maximisation)

- A unified approach:

$$\max_t \left[U(t) + V(B + R(t) - C(t))\right] , \tag{29}$$

where $V(£)$ now measure the value of the next best use of money, rather than investing in playing talent.
The first order condition is:

$$U'(t) = V'(B + R(t) - C(t))(C'(t) - R'(t)) . \tag{30}$$

- V' measures the marginal utility of an extra unit of money.

- $C'(t) - R'(t)$ measures the net marginal cost of an extra unit of talent for the team.

- The FOC can be re-written as:

$$MR(t) + \frac{U'(t)}{V'(B + R(t) - C(t))} = MC(t) \; . \tag{31}$$

Now the team hires up to the point where the marginal cost of talent is equal to the marginal revenue (the pecuniary value) plus the marginal utility to the owner from talent (non-pecuniary), with the latter measured in money terms, i.e. V' gives the effective exchange rate between money and utility.

- This unified objective function allows us to add other new features to the model, without having to disentangle competing objectives: i.e. we don't have to consider separate cases of profit and win-percent maximisers.

2. Contested and uncontested talent

- The Open model presents the market for talent as if teams are buying from different shops.

- The Fixed-pool model assumes that all talent is contested.

- B&S extend this to a labour market where clubs decide which players to approach and contest.

- The teams' choice object is a vector of wage offers to the available talent.

- In equilibrium, these offers are a best response to those of the other teams in the market.

- In this way, the talent market is modelled like an auction.

- In this extension, some of the talent will be uncontested, and then only receive their reservation wage level.
- It is endogenously determined by the teams which talent (players) are contested or uncontested.
- When talent supply is inelastic, this extension replicates the ‘Coasian’ result for allocation from the fixed-pool model, but with a more realistic sports labour market.
- Results can be generalised to heterogeneous talent (players).

Note: *I do not expect you to learn how the results from the 2nd extension in Burguet and Sakovics (2018) are derived and proved. But you should be able to write about what their contribution was, relative to the ‘traditional’ models.*

A practice exam-type problem

There are two teams in a league, A and B. These teams are profit maximisers and they compete over a fixed talent pool. The total amount of talent in this leave is given by $t_A + t_B = 1$. Talent is paid a market wage rate w. The teams' revenue functions are given by:

$$R_i = m_i(t_i - t_i^2/2), \quad i \in \{A, B\}, \quad \{m_A, m_B\} = \{1, 2\} \ .$$

Continued:

(i) Briefly discuss why the market for talent in this league can be described as ‘Coasian’.

(ii) What is the equilibrium allocation of talent between teams A and B, and what is the market price for talent?

Continued...

Continued:
The sports talent in this league believe they are not benefiting enough from it. They get together and buy out the owners of the league. Now they can change the rules to their own benefit. They still receive market wages from the teams as well. The talent are deciding which of two rule changes to impose on the teams.

1: They own the league, so they should receive and keep a share $\beta \in (0,1)$ of the teams' revenues.
2: The team's should pay a minimum wage of $9/10$.

(iii) Using a graphical representation of the equilibrium in this market for talent, and words, discuss the impacts of each proposed rule change on competitive balance, the salaries of the talent, and the total pecuniary benefits from the league received by the talent.

Outline answer:

(i) ...

(ii) Both teams are profit maximisers, so hire talent up to the point where marginal revenue is equal to marginal cost. This implies:

$$(1 - t_A) = w \; . \tag{32}$$

$$2(1 - t_B) = w \; . \tag{33}$$

Finally, a fixed talent pool means that

$$t_A + t_B = 1 \; . \tag{34}$$

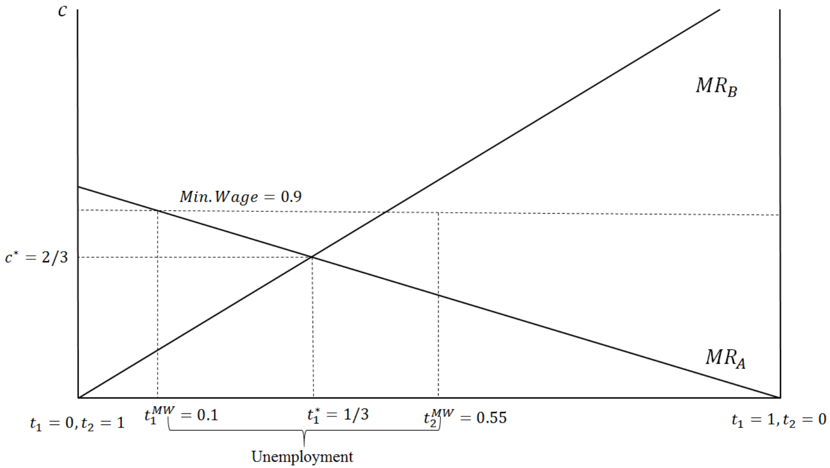
Solving these three equations simultaneously, the equilibrium is given by $\{t_A^*, t_B^*, w^*\} = \{1/3, 2/3, 2/3\}$.

Continued:

(iii) First, consider rule change 1. This is equivalent to the revenue sharing extension of the model discussed in the lecture, except the clubs are in effect now sharing part of their revenues with the talent. Therefore, you could answer this question by providing a variant of the graphical representation from before. Since the model is 'Coasian', there will be no effect on the allocation of talent between A and B, and so no effect on competitive balance. The market wage rate and salary of the talent will decrease by a factor $1-\beta$, or an amount βc^{π}. But they will now get β of the total league revenues, which are unchanged. Since the teams made a profit, the talent overall benefits from the rule change by an amount $\beta(R_1+R_2-c)^{\pi}$.

Continued:
Second, consider rule change 2. The minimum wage is above the market clearing rate. The talent pool may be fixed, but now there is going to be unemployment. Team A will hire up to the minimum wage, so $\tilde{t}_A = 0.1$. Similarly, $\tilde{t}_B = 0.55$. So, the unemployment rate is 0.35. So, 0.65 of the talent benefit from an increase in their salary, with the overall salary bill in the league decreasing from 2/3 to 0.585. But 0.35 of the talent lose 2/3. The competitive balance in the league also suffers. The poorer team, A, is hit hardest by the minimum wage. See graphical representation on next slide:

Practice exam Q - impact of a minimum wage.



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