Economics of Sport (ECNM 10068)

Lecture 7: Theories of a League - Part I

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Theories of a League - Part I

Issues covered:

- How to design a league (tournament)?

- A theoretical model of resource allocation in sports leagues

Main reading:
Chapter 2, Dobson-Goddard "The Economics of Football" 2nd ed. Cambridge 2011; Noll, R. (2003). "The Organization of Sports Leagues", *Oxford Review of Economic Policy*, 19(4), pages 530-551.

What is a league?

- Jointly organised set of bilateral matches

Why set one up?

- To increase interest in the outcome of competition
- Lower transaction costs (centralised organisation)
- Standardise rules and enforce discipline (e.g. financial fair play)
- Easier to market your sport as a package, especially for the sale of broadcasting rights

Two examples of new/future leagues in major professional sports
What motivated their creation and design?

UEFA Nations League (international football):
https://www.uefa.com/... , News article

Test Schedule / League (international cricket):
https://www.icc-cricket.com... , News article

Is this the most complex real-world league design to find a winner?
America's Cup (sailing; incl. qualification stages): 2017 results
Why is it so complex?

League formats: Round-robin

- Let there be n teams (players), who must each play against one another head-to-head exactly once.
- Therefore, the league will have a total of $\frac{n(n-1)}{2}$ matches.
- The league can be described by a series of rounds $k = I, II, III, IV \dots.$
- The total number of rounds is $n-1$ if n is even, and n if n is odd. In the latter case each team has one round in which they have a bye (no game).
- Final positions are determined by some cumulative metric from the games played (e.g. points for wins or draws).
- In case of a tie in the final standings, some other metric could be used to rank teams (e.g. goal difference, net run rate); or could use head-to-head record; or teams could play a tie-breaker game.

Round-robin tournament - an illustration
Home and Away (2 matches against each other team)

		TEAM						Round-robin 1 (or vertical Home)
		1	2	3	4	5	6	
TEAM	1		I (0-0)	II (2-0)	III (1-2)	IV (0-6)	V (3-3)	
	2	X		I (1-0)	V (1-1)	III (2-0)	IV (4-1)	
	3	IX	VI		IV (2-2)	I (1-2)	II (3-1)	
	4	VII	IX	VI		V (3-0)	II (12-11)	
	5	VIII	VII	X	VI		I (0-0)	
	6	VI	X	VIII	IX	VII		
	Round-robin 2: (or vertical Away)							

Source: own imagination; key: (x-y) gives score of game between (Home-Away)

Direct elimination

- Round-robin leagues require a lot of games and time.
- Towards the end, they can lead to many games which are irrelevant for determining the eventual league champion.
- A quicker way to find a winner is to use direct elimination.
- Teams lose and they are out of the tournament (no draws allowed): makes games more important/exciting.
- If there are n teams, then need to stage $\log_2 n$ rounds, rounded up to nearest integer: e.g. 32 teams implies 5 rounds ($2^5 = 32$) - but 20 teams also requires 5 rounds.

How many games in a direct elimination tournament?

- Let r be the number of rounds - number of matches is given by:

$$N = n - 2^{r-1} + 2^{r-2} + \ldots 2^{r-r} \ .$$

  Use

$$2N = 2n - 2^{r} + 2^{r-1} + \cdots + 2^{r-(r-1)},$$

  then

$$N = 2N - N = n - 1$$

- But final results (especially below 1st place) depend on “luck of the draw”; i.e. who players get drawn against in each round.
- This might encourage greater participation or interest in some contexts, but generally, fans (demand) want to see the best players reaching the finals.

Partial solutions:

1. “Seeding” (e.g. Snooker, Darts, Tennis) - higher ranked players are kept apart until the latter stages
2. Many tournaments or multiple games (matches) in each round - luck averages out
3. Repêcharge, double elimination, a “Plate” trophy

League hierarchy

- Promotion and relegation (members entering or dropping out of a league): can significantly affect incentives; e.g. European football.

- The presence of promotion/relegation is likely to increase short-termism in member investment and resource allocation decisions.

- But competition must be good for efficiency...?

- ‘US-style’ closed leagues (to new members) are inefficient.
- They effectively create a ‘home territory’ for their members, turning them into local monopolies.
- Despite a lack of economic competition, this can still be a successful model: generates large and relatively secure (predictable) streams of profits for members, allowing long-term investment decisions (Stadium, facilities and youth development - improving fan and player welfare).

Are monopoly sports leagues /teams ‘natural’?

- The most common defence for the ‘territorial’ rights of teams and leagues is that competition would create an unstable market.

- The argument is that local sports markets are ‘winner takes all’ anyway, so why destabilise the natural monopoly by introducing competition.

- This argument does not stand up to careful scrutiny:

- First, why have barriers to competition in the first place then? Even short-term but doomed competition is likely to bring benefits to consumers.

- Second, there are too many counter-examples of successful sports markets without closed entry and/or territorial rights.

Competition for talent/resources when the pool is fixed

A model of an n-team league:

- There are n teams, denoted by $i \in \{1, 2, ..., n\}$.

- They hire talent t_i, where the pool of talent is **fixed** (closed), i.e. $\sum_i^n t_i = T = 1$ (normalisation).

- A team i's revenue depends on three factors:
  1. Market size: m_i
  2. The overall quality of league standards: $\bar{t} = T/n = 1/n$.
  3. The relative quality of team i: $\tau_i = t_i/\bar{t} = nt_i$

- The price/marginal/average cost of talent is constant, c.

- The talent choice of team i affects the quantity hired by the other teams.

- The revenue of team *i* is given by:

$$R_i = \eta m_i(\bar{t} - \theta \bar{t}^2)(\tau_i - \xi \tau_i^2) \tag{1}$$

$$= \frac{\eta m_i (n - \theta)(t_i - n\xi t_i^2)}{n} \ . \tag{2}$$

The equilibrium depends on the objectives of the teams:

- Profit maximisation: choose talent up to the point where marginal revenue = marginal cost; $MR_i = MC_i = c$

- Win-percent maximisation, with zero-profit constraint: choose talent up to the point where average revenue = average cost; $AR_i = AC_i = c$.

- With a fixed pool, we assume that c is determined endogenously.

What is the win percent?

- Standard in literature to assume that the win percent for team i in games against j is given by $w_i = t_i/(t_i + t_j)$.

- This gives a total win percent in an n-team league, where teams play each other the same number of times, of $\sum_{j \neq i} [t_i/(t_i + t_j)] / (n - 1)$.

- But in a two team league, this is simplified, since $w_i = t_i = \tau_i/2$.

The 2-team fixed talent pool model ($n = 2$)

- Revenue functions given by:

$$R_i = \frac{\eta m_i(2-\theta)(t_i - 2\xi t_i^2)}{2} = \psi m_i(t_i - \phi t_i^2) \ , \qquad (3)$$

  where $\psi = \frac{\eta(2-\theta)}{2}$ and $\phi = 2\xi$.

- Assume both teams are profit maximisers.

- The equilibrium values of $\{t_1^*, t_2^*, c^*\}$ are determined by three equations:

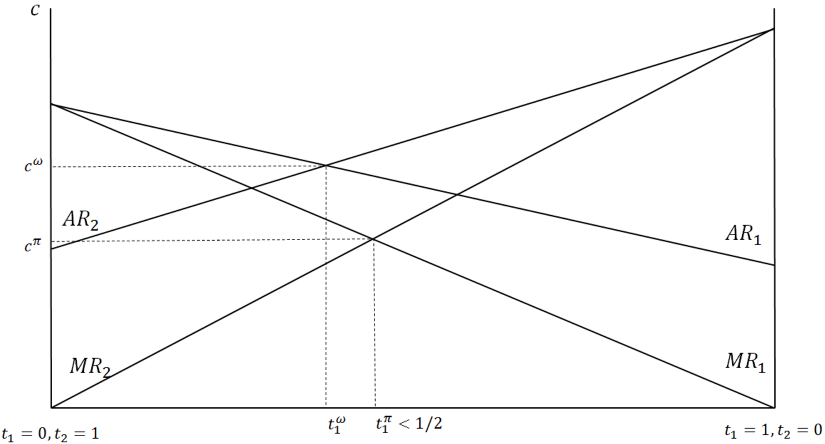
$$MR_1 = \psi m_1(1 - 2\phi t_1^*) = c^* \ , \tag{4}$$
$$MR_2 = \psi m_2(1 - 2\phi t_2^*) = c^* \ , \tag{5}$$
$$1 = t_1^* + t_2^* \ , \tag{6}$$

with solution:

$$c^* = \frac{\psi m_1 m_2 (2 - 2\phi)}{m_1 + m_2} \ , \tag{7}$$
$$\frac{t_1^*}{t_2^*} = \frac{m_1 - m_2 + 2\phi m_2}{m_2 - m_1 + 2\phi m_1} \ . \tag{8}$$

- We can represent this graphically as follows:

Equilibrium - 2-team fixed talent pool - $m_2 > m_1$



Source: lecturer

Equilibrium - 2-team fixed talent pool - continued

- As drawn, $m_2 > m_1$, therefore $t_2^* > t_1^*$.
- The relative difference in talent is larger if the teams are both win-percent rather than profit maximisers: $\frac{t_1^\pi}{t_2^\pi} > \frac{t_1^\omega}{t_2^\omega}$, which means that competitive balance is also reduced.
- The total/marginal/average cost of talent (player salaries), c^* is greater if teams are win-percent maximisers.

Extensions of the model (next lecture)

- What if teams don't need to break even, and still want to win at all costs?
- What effect does a revenue sharing model have in a sports league?
- What does a payroll cap do?
- Can we model endogenous bidding/competing for talent, rather than exogenously assume it with a 'fixed' pool.

A practice exam-type problem

There are two football teams in a city, United and City, whose revenues only depend on the matches they play against one another. Success in these matches is only determined by the relative quality of the players in each team. The two teams hire a quantity of playing talent, t_i, from a fixed pool of size 1. The profits of each team are given by:

$$\Pi_i = m_i(t_i - \frac{t_i^2}{2}) - wt_i \geq 0, \qquad i \in \{Utd, City\} ,$$

where the market power of the teams is given by $m_{Utd} > m_{City} > 0$. The wage rate paid to the playing talent is given by w, and is endogenously determined. United is a profit maximiser. City is a ‘win-percent’ maximiser.

Continued:

(i) Briefly discuss why it is generally more realistic to model a sports league with a fixed talent/resource pool, a 'closed' league, rather than without such a constraint, an 'open' league.

(ii) What is the equilibrium ratio of talent in the United team compared with City? What is the equilibrium wage rate paid to the playing talent? Your answers should be functions of m_i.

(iii) Now assume that $m_{City} = 1$. For what value of m_{Utd} is the football played in this city perfectly competitive?

(iv) Assume that initially the equilibrium is in perfect competitive balance, with values of m_i as per your answer to (iii). Graphically and in words, describe the impact on this initial equilibrium if City suddenly had access to a large amount of cash, B, with which it finances the hiring of playing talent.

Outline answer:

(i) ...

(ii) United is a profit maximiser, so hires talent up to the point where marginal revenue is equal to marginal cost. This implies:

$$m_{Utd}(1 - t_{Utd}) = w \ . \tag{9}$$

City is a win-at-all-costs team, though it must still break even. Therefore, it hires talent up to the point where average revenue is equal to average cost. This implies:

$$m_{City}(1 - \frac{t_{City}}{2}) = w \ . \tag{10}$$

Finally, a fixed talent pool means that

$$t_{Utd} + t_{City} = 1 \ . \tag{11}$$

Solving these three equations simultaneously, an equilibrium for this league (talent market) is given by:
$\{t^*_{Utd}, t^*_{Cty}, w^*\} = \{\frac{2m_{Utd}-m_{Cty}}{2m_{Utd}+m_{Cty}}, \frac{2m_{Cty}}{2m_{Utd}+m_{Cty}}, \frac{2m_{Utd}m_{Cty}}{2m_{Utd}+m_{Cty}}\}$, and

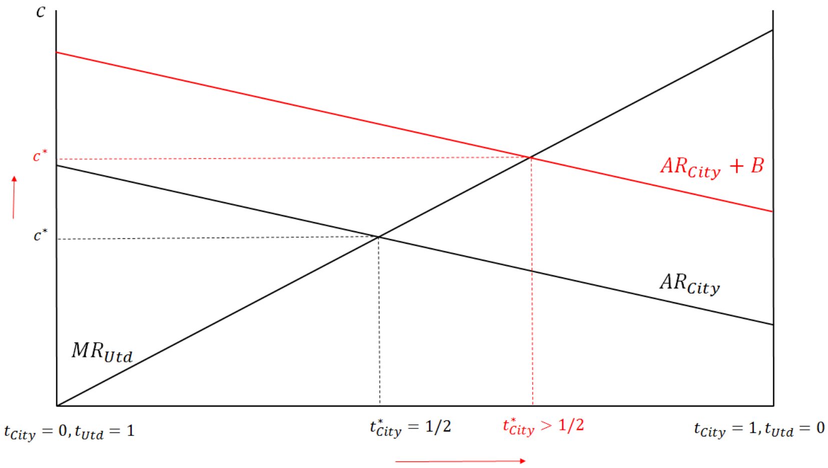
$$\frac{t^*_{Utd}}{t^*_{City}} = \frac{m_{Utd}}{m_{Cty}} - \frac{1}{2} \,.$$

So in the equilibrium, United will have a relatively better team and more success because of $m_{Utd} > m_{City}$. But the fact that City have a win-at-all-costs mentality means that their market power disadvantage might not be enough to guarantee United actually have the better team.

(iii) Using the answer from (ii), it is clear that for competitive balance, $\frac{t^*_{Utd}}{t^*_{City}} = 1$, requires that United have 50% greater market power than City: $m_{Utd} = 1.5$

(iv) ... & see next slide
[Please make sure you draw graphical answers (theoretically) accurately! This does not mean you will be penalised if lines are not perfectly straight, nor if slopes are not exactly equal to x. But you would be penalised if, in this example, the MR_{Utd} was not drawn going through the point $t_{City} = 0, t_{Utd} = 1$, or if MR_{Utd} was not clearly steeper than AR_{City}].

The impact on a perfect-competitive-balance equilibrium of one team getting a cash windfall



Source: lecturer