Economics of Sport (ECNM 10068)

Lecture 1: Introduction; The Theory of Contests

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Introduction

Can studying the wide context of competitive sports provide new insights into economic behaviour?

Can economic theory...

- ... help to predict the outcomes of sporting competition?
- ... be used to design (in some sense) optimal sets of rules and structures/leagues/tournaments?
- ... make sense of why the market for sport and its supply-chain are quite different from any other market?

Main reading:

Chapter 1, Dobson-Goddard “The Economics of Football” 2nd ed. Cambridge 2011;
S. Szymanski. (2003) “The Assessment: The Economics of Sport”, *Oxford Review of Economic Policy*, (19)4: 467-477, doi:10.1093/oxrep/19.4.467.

Lecture 1 - The Theory of Contests

Issues covered:

- What does an economic model of sporting competition look like?

- When each player can observe the actions of the other players, what is an equilibrium?

Main reading:

S. Szymanski. (2003). “The Assessment: The Economics of Sport”, *Oxford Review of Economic Policy*, (19)4: 467-477, doi:10.1093/oxrep/19.4.467;
Dietl, H., Franck, E., Grossmann, M. and Lang, M. (2012). “Contest Theory and its Applications in Sports”, The Oxford Handbook of Sports Economics Volume 2, edited by S. Shmanske and L. Kahane. New York, USA: Oxford University Press. doi:10.1093/oxfordhb/...

Contest Theory in Economics more generally

- Rewards or outcomes based on individual's relative performance or outputs rather than absolute amounts.

- Seminal work studied the optimal design of rent-seeking contests for public funds; but extended to other contexts such as the labour market (firms competing for workers).

- Generally related to the wide literature on auction theory, and their optimal design.

Contest Theory in the Economics of Sport

- In sporting contests there is usually some uncertainty about who will win: the player who puts the most into the competition can still lose.

- Some sports are very discriminating - the best player almost always wins: athletics, swimming, boxing.

- In others, even the best player wins rarely: golf, international team sports (e.g. football World Cup).

- In a standard auction only the winning bidder pays - but in sporting contest everybody ‘pays’ before finding out who wins.

- Sporting contests can therefore be modelled as imperfectly discriminating all-pay auctions.

A simple model of sporting contest

- Let each player (team) be denoted by $i(j) = \{1, 2, ..., N-1, N\}$, such that there are N players in total.
- Each player faces a 'contest success function': gives the probability of success p_i for individual i, depending on the amount of effort (resources) they put in $x_i \geq 0$, relative to the amount of effort put in by others $x_j \geq 0$.
- One example is a logit function:

$$p_i = \frac{x_i^\gamma}{\left(\sum_{j \neq i}^{N-1} x_j^\gamma\right) + x_i^\gamma}, \quad \gamma \geq 0, \quad \sum_i^N p_i = 1. \tag{1}$$

- γ measures how discriminatory the effort put in is:
    - $\gamma \to 0$: $p_i = 1/N$, $\forall i$; effort is irrelevant
    - $\gamma \to \infty$: $p_i = 1$ if $\{x_i > x_j,\ \forall i \neq j\}$, and $p_i = 0$ otherwise; everybody pays, but the highest 'bidder' wins, making this an 'all pay auction'.

- Define a payoff (revenue) function from the contest as:

$$\pi(x_i, x_{-i}) = p_i(x_i, x_{-i})V_i - c_i(x_i) + R_i \geq 0 \quad (2)$$

  - $\pi(x_i, x_{-i})$: net payoff (profit) given own and others' efforts
  - $p(x_i, x_{-i})$: prob. of success given own and others' efforts
  - V_i: Prize from winning contest
  - $c_i(x_i)$: Cost of effort
  - $c_i'(x_i) > 0$, $c_i''(x_i) \geq 0$: Cost is increasing in effort, and perhaps increasingly so
  - R_i: Fixed value (could be negative) from taking part.
- Key assumption: own and others' actions are always observed.
- Potential to allow for many different types of asymmetry in the model, for example: $V_i, c_i(x), R_i$, γ_i

What is the optimal choice of effort for player i, taking as given the possible actions of others?

- Substitute (1) into (2). Find the best response to any possible effort choice of other players by the first order condition (FOC), i.e. maximising w.r.t. x_i:

$$c_i'(x_i) = \gamma V_i x_i^{\gamma-1} \frac{\sum_{j\neq i}^{N-1} x_j^{\gamma}}{\left(\left(\sum_{j\neq i}^{N-1} x_j^{\gamma}\right) + x_i^{\gamma}\right)^2} \tag{3}$$

- If it exists, a Nash Equilibrium (NE) in pure strategies $\{x_1^*, \ldots x_N^*\}$ would be characterised by the intersection of these best response functions.
- Solving this looks like it could get messy ...

The Symmetric Equilibrium

Assume players are symmetric (ex ante homogeneous):

- Can assume (guess) that the equilibrium is given by unique pure strategies $x_i^* = x^*$, $\forall i$. Substitute this into (3) to find:

$$x^* = \frac{\gamma V(N-1)}{c'(x^*)N^2} \tag{4}$$

- Optimal eq. effort is increasing in the amount of discrimination and the size of the prize
- Effort is decreasing in the number of players and marginal cost *[Note, could simplify here by letting $c(x) = cx$, so $c'(x^*) = c$, i.e. a constant marginal cost.]*
- In this eq., $p_i^* = p^* = 1/N$, $\forall i$

But... we need to check that this symmetric pure strategy eq. exists.
First:

- Does it satisfy the participation constraint (individual rationality), $\pi(x^*) \geq 0$?

  [Assume for simplicity $c(x) = c$*]*

  Participation then requires $x^* \leq V/cN + R/c$.
  Using (4), this is the same as $R \leq \frac{V(N-\gamma(N-1))}{N^2}$.
  Which simplifies further when we assume $R = 0$ to $\gamma \leq N/(N-1)$.
  - If $\gamma > N/(N-1)$, with symmetric players, it could be that nobody plays, and there is no contest.
  - With heterogeneity, some players might not play, but the contest could still go ahead with an eq. in pure strategies for those that do play.

Second:

- With the contest set-up here, when $N \geq 3$, we need to check that $\pi(x_i, x_{-i})$ is concave:

$$\frac{\partial^2 \pi(x_i, x_{-i})}{\partial x_i^2} < 0 \tag{5}$$

- For $c(x) = c$, the existence of a symmetric pure-strategy eq. requires $\gamma < N/(N-2)$.
- If γ is larger, there could be mixed strategy equilibria.

Can we test the theory?

- If we can observe the elements of (4), then it motivates a regression model to test the theory.
- Taking logs, it gives us something linear we can then estimate using a least squares regression.
- For example, to test the elasticity of effort to the size of the prize (β_1) across a number of contests k, for a set of players i:

$$\log(x_{ik}) = \beta_0 + \beta_1 \log(V_k) + \beta_2 \log((N_k - 1)/N_k^2) + \varepsilon_{ik}$$

An example of a mixed-strategy equilibrium, with symmetric players and $\gamma = \infty$.
Assume $c(x) = c$ *and* $R = 0$*.*

- Let $P(x)$ be the cumulative density function of the mixed strategy played by all, with $x \in [\underline{x}, \overline{x}]$.
- With symmetric players, the probability of individual i being successful is then $P(x)^{(N-1)}$; i.e. the other $N-1$ players when mixing have lower effort levels.
- For a mixed-strategy eq. of this type to exist with $P(x)$ it must be the case that

$$VP(x)^{(N-1)} - cx = VP(\underline{x})^{(N-1)} - c\underline{x} = 0, \; with \; \underline{x} = 0. \qquad (6)$$

- Therefore $P(x) = \left(\frac{cx}{V}\right)^{\frac{1}{N-1}}$. And $P(x) = 1$ implies $\overline{x} = V/c$, i.e. players break even when sure to win.

Is it realistic that V and R are exogenous?: so far they do not depend on x.

- In reality, the revenues and thus prizes in sport could depend on how competitive the games are.
- One possible measure of *competitive balance* in a contest is given by $CB = \prod_i^{N'} p_i^*$, where $N' \leq N$ is the number of players who participate in the equilibrium.
- With $N' = 2$, $CB = p_1^*(1-p_1^*)$. Which is maximised when $p_1^* = 1/2$. More generally, for N' participating players, CB is maximised when $p_i^* = 1/N', \quad \forall i$.
- More on this in the next lecture, when we discuss the Demand for Sport.

A practice exam-type problem

A wealthy businessman invites two teams called E and S to a Caribbean island to play a one-off, winner-takes-all, game (of cricket) with a large prize $\$V$.[2]

Both teams face the same success function: $p_i = \frac{x_i}{x_i + x_j}$, where x_i is the resources that the teams put in to playing the tournament. Team S has a constant marginal cost equal to 1. Team E has a constant marginal cost equal to $\alpha \geq 1$. Team E also faces a fixed cost of taking part R, but team S faces no fixed costs. In all other ways the two teams are identical.

(i) Write down the objective problems of the two teams, taking account of how they depend on the actions of the other team.

(ii) Assuming V is very large [i.e. a contest definitely takes place], what are the equilibrium values of the inputs $\{x_E^*, x_S^*\}$ and the probabilities of success $\{p_E^*, p_S^*\}$ as functions of the parameters V, α, R?

[2] This question is partially motivated by the (controversial) 2008 Stamford Super Series cricket tournament between ***E***ngland and the ***S***tamford Superstars.

Continued...

(iii) Using your answer to part (ii), discuss how 'competitive balance' is affected by the magnitude of α.

(iv) Now suppose $\alpha = 2$ and $R = \$100,000$. How large does V then need to be such that team E will play the game?

Outline Answer:

[In an exam, you should explain each step and add brief insights on the economic meaning of what is being described by the problem]

(i) For team E:

$$\max_{x_E \geq 0} \frac{x_E}{x_E + x_S} V - \alpha x_E - R \ \ (\geq 0). \tag{7}$$

For team S:

$$\max_{x_S \geq 0} \frac{x_S}{x_E + x_S} V - x_S \ \ (\geq 0) \tag{8}$$

Continued...

(ii) FOC for team E:

$$\frac{Vx_S}{(x_E + x_S)^2} - \alpha = 0 \tag{9}$$

FOC for team E:

$$\frac{Vx_E}{(x_E + x_S)^2} - 1 = 0. \tag{10}$$

The pure-strategy Nash eq. is given by the intersection of these best response functions. Combining (9) & (10) gives $x_S^* = \alpha x_E^*$. Substitute this back into (9) to show:

$$\{x_E^*, x_S^*\} = \{\frac{V}{(1+\alpha)^2}, \frac{\alpha V}{(1+\alpha)^2}\},$$

And so,

$$\{p_E^*, p_S^*\} = \{\frac{1}{1+\alpha}, \frac{\alpha}{1+\alpha}\},$$

Continued...

(iii) A measure of competitive balance is given by:

$$p_E^* p_S^* = \frac{\alpha}{(1+\alpha)^2}. \tag{11}$$

When $\alpha = 1$, $p_E^* p_S^* = 1/4$. But for any $\alpha > 1$, team E is at a competitive disadvantage, and the measure of balance is strictly decreasing in α. Formally,

$$\frac{\partial \left[p_E^* p_S^*\right]}{\partial \alpha} = \frac{1-\alpha}{(1+\alpha)^3} < 0 \quad \mathit{if}\ \alpha > 1. \tag{12}$$

Continued...

(iv) The participation constraint for team E is given by:

$$R \leq Vp_E^* - \alpha x_E^* \tag{13}$$

For $\alpha = 2$ and $R = \$100,000$, this constraint binds with equality if $V = \$900,000$.